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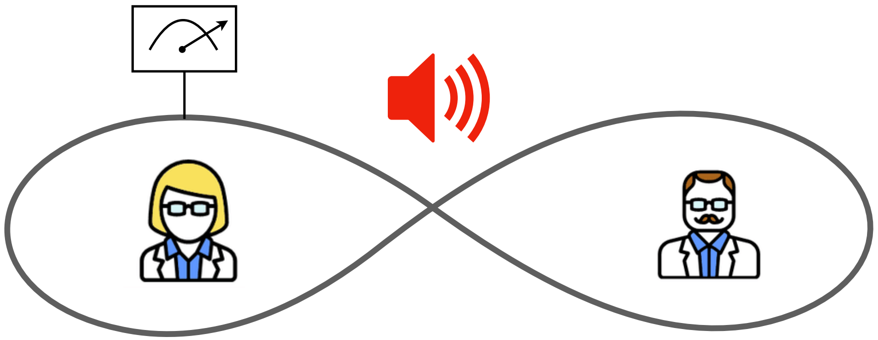
Alice, Bob and the quantum ice-cream

Quantum Talent Symposium, Munich

Sara Murciano

17 September 2024



Entanglement: from an obstruction to a resource

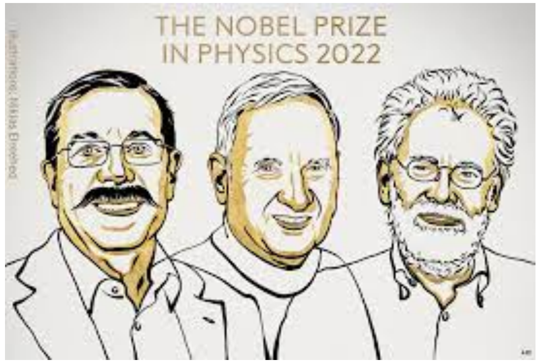
‘30s

‘60s

last decades

spooky action
at distance

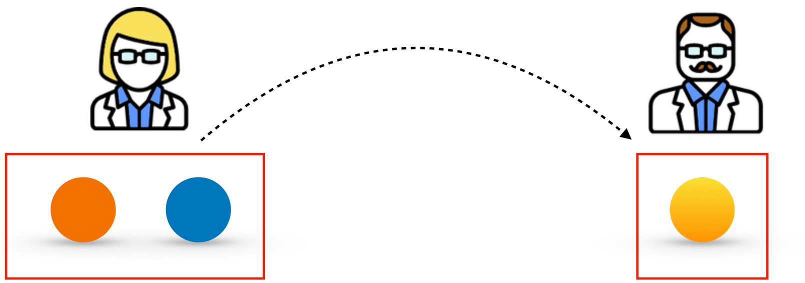
Bell inequalities



Entanglement: from an obstruction to a resource

'30s '60s last decades

Resource for quantum technology, communication and information



Quantum Teleportation

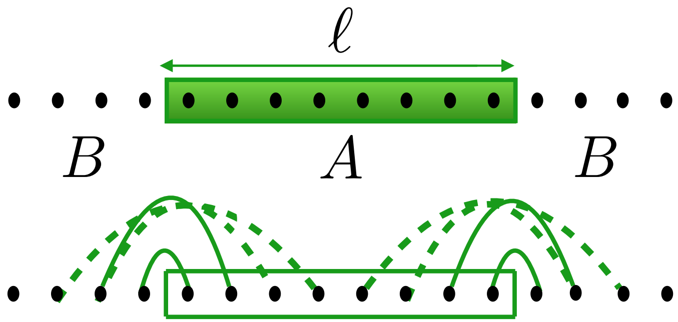
M. A. Nielsen, I. L. Chuang, Cambridge University Press (2010)

Entanglement: from an obstruction to a resource

Detect and describe phase transitions

ground state of a local gapped Hamiltonian

$S_A \propto$ Area separating A and B



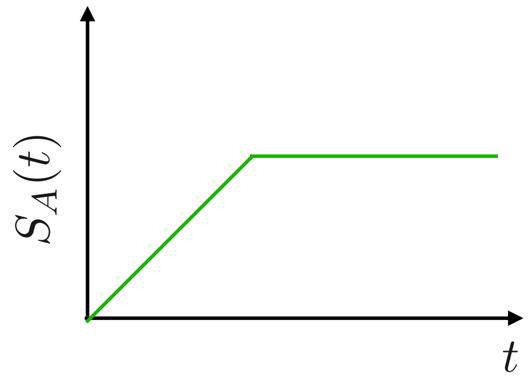
1+1D critical theories:

$S_A = \frac{c}{3} \log \ell$ (**universal** properties)

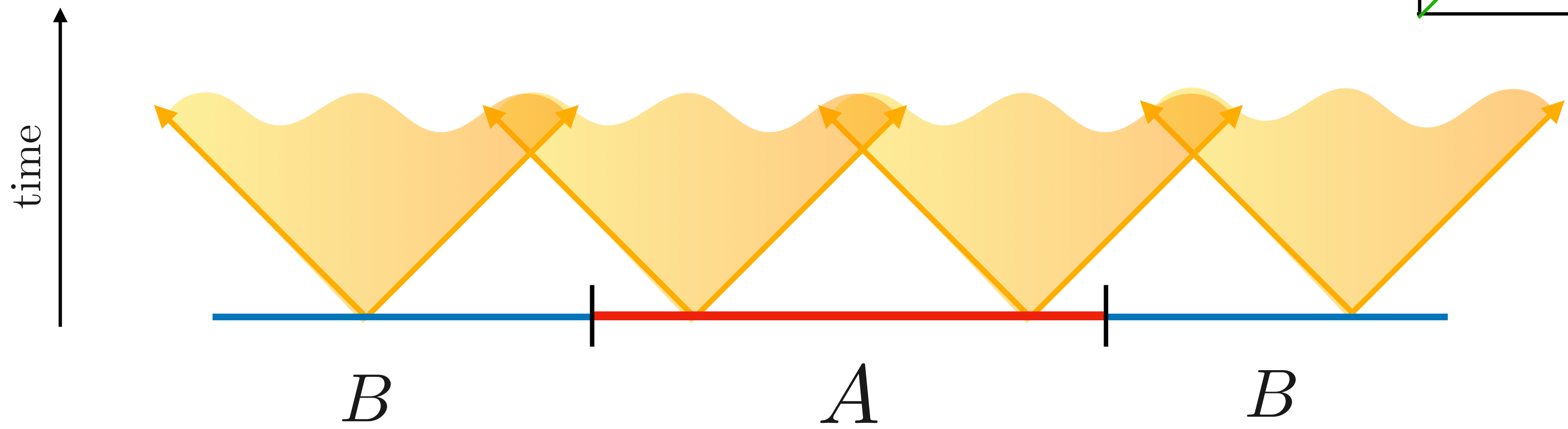
M. B. Hastings, Phys. Rev. B 73, 085115 (2006)
C. Holzhey, F. Larsen, F. Wilczek, Nucl. Phys. B 424, 443 (1994).

Entanglement: from an obstruction to a resource

Connection between entanglement and
thermodynamics in non-equilibrium systems



Integrable systems:
a **quasiparticle** picture



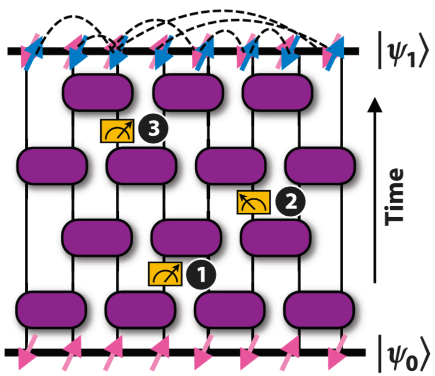
M. Rigol, V. Dunjko, M. Olshanii, Nature 452, 854 (2008)
M. Rigol, V. Dunjko, V. Yurovsky, M. Olshanii, Phys. Rev. Lett. 98, 050405 (2007)

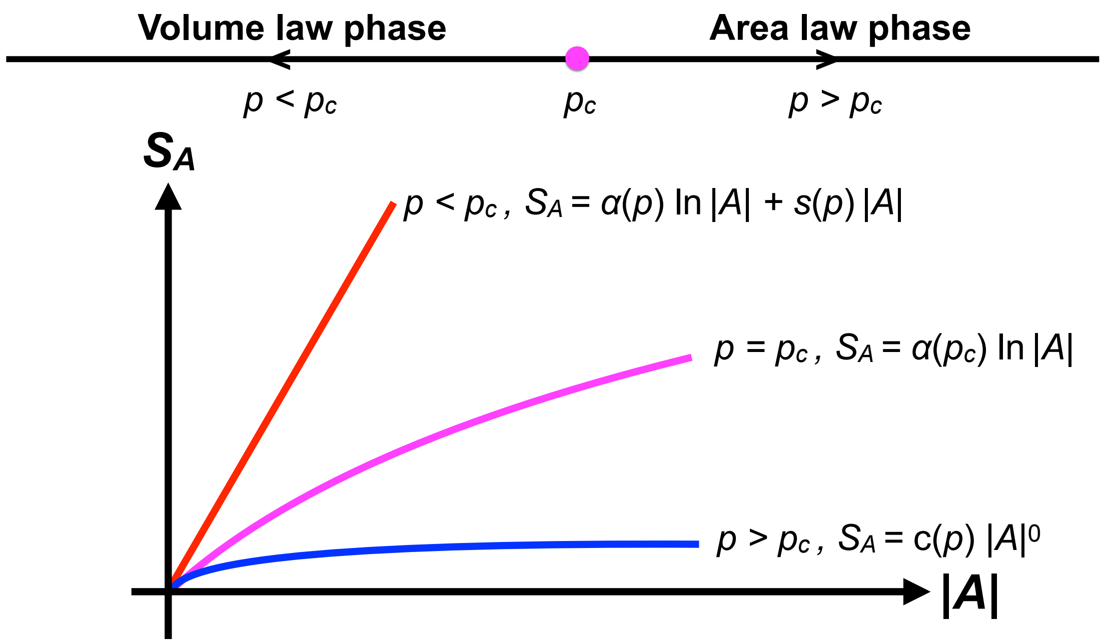
P. Calabrese, J. Cardy, J.Stat.Mech. 0706:P06008, 2007
V. Alba, P. Calabrese, PNAS 114, 7947 (2017)

Entanglement: from an obstruction to a resource

'30s '60s last decades

Measurement-induced phenomena





Novel quantum phenomena:
entanglement transitions

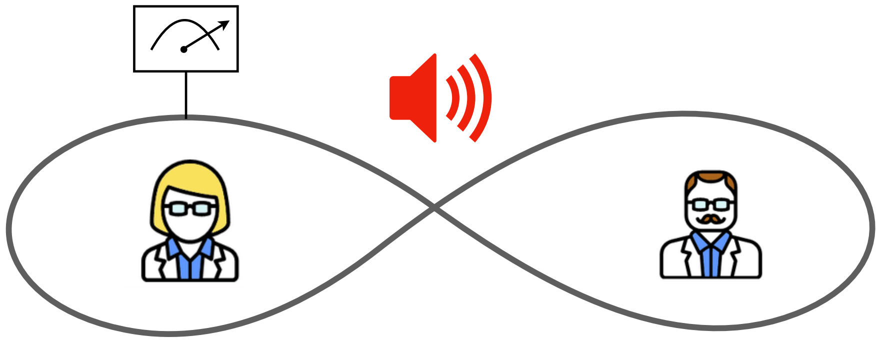
reviewed in M. Fisher, V. Khemani, A. Nahum, S. Vijay, Annu. Rev. Condens. Matter Phys. 2023. 14:335-79

Y. Li, X. Chen, M. P. A. Fisher, Phys. Rev. B 98, 205136 (2018).

In this talk:

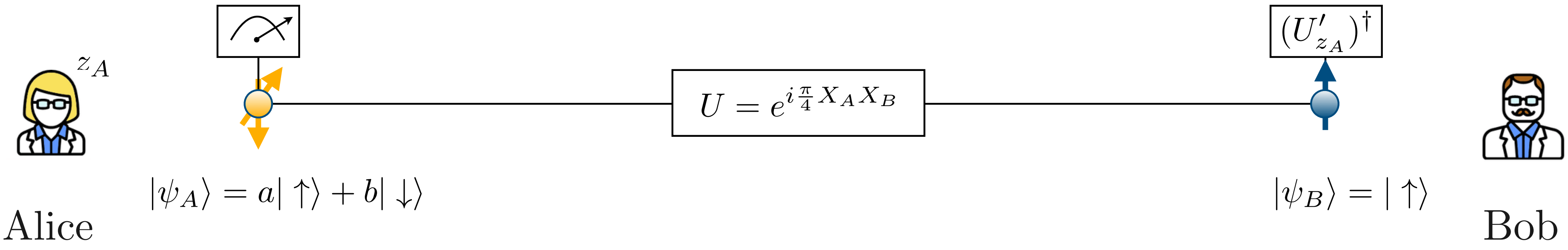
‘30s — ‘60s — last decades — today

Alice, Bob and the quantum ice-cream



Quantum teleportation

Goal: Transfer Alice's state to Bob without revealing her precise wavefunction



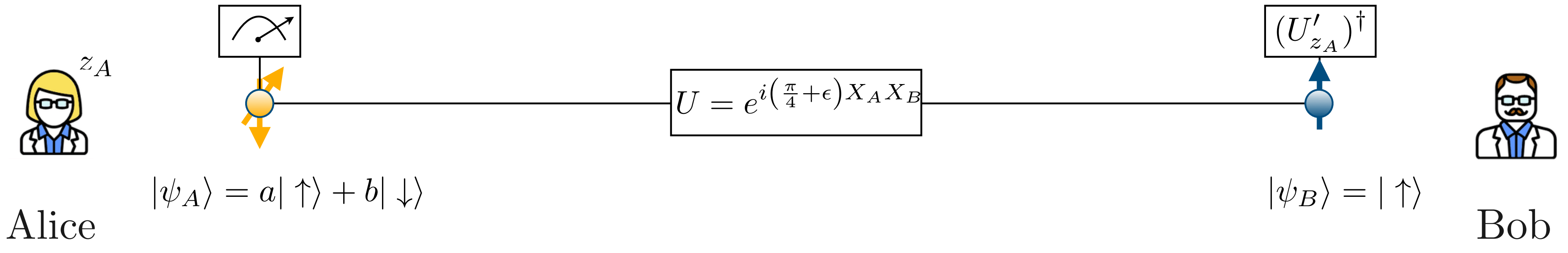
$$|\psi_A\rangle|\psi_B\rangle \xrightarrow[\text{Entangle}]{} U|\psi_A\rangle|\psi_B\rangle \xrightarrow[\text{Alice measurement with outcome } z_A]{} |\psi_{z_A}\rangle = U'_{z_A}|\psi_A\rangle \xrightarrow[\text{"undo" unitary}]{} |\psi_A\rangle$$

Transfer complete!

But…no one is perfect.

Quantum teleportation

Goal: Transfer Alice's state to Bob without revealing her precise wavefunction

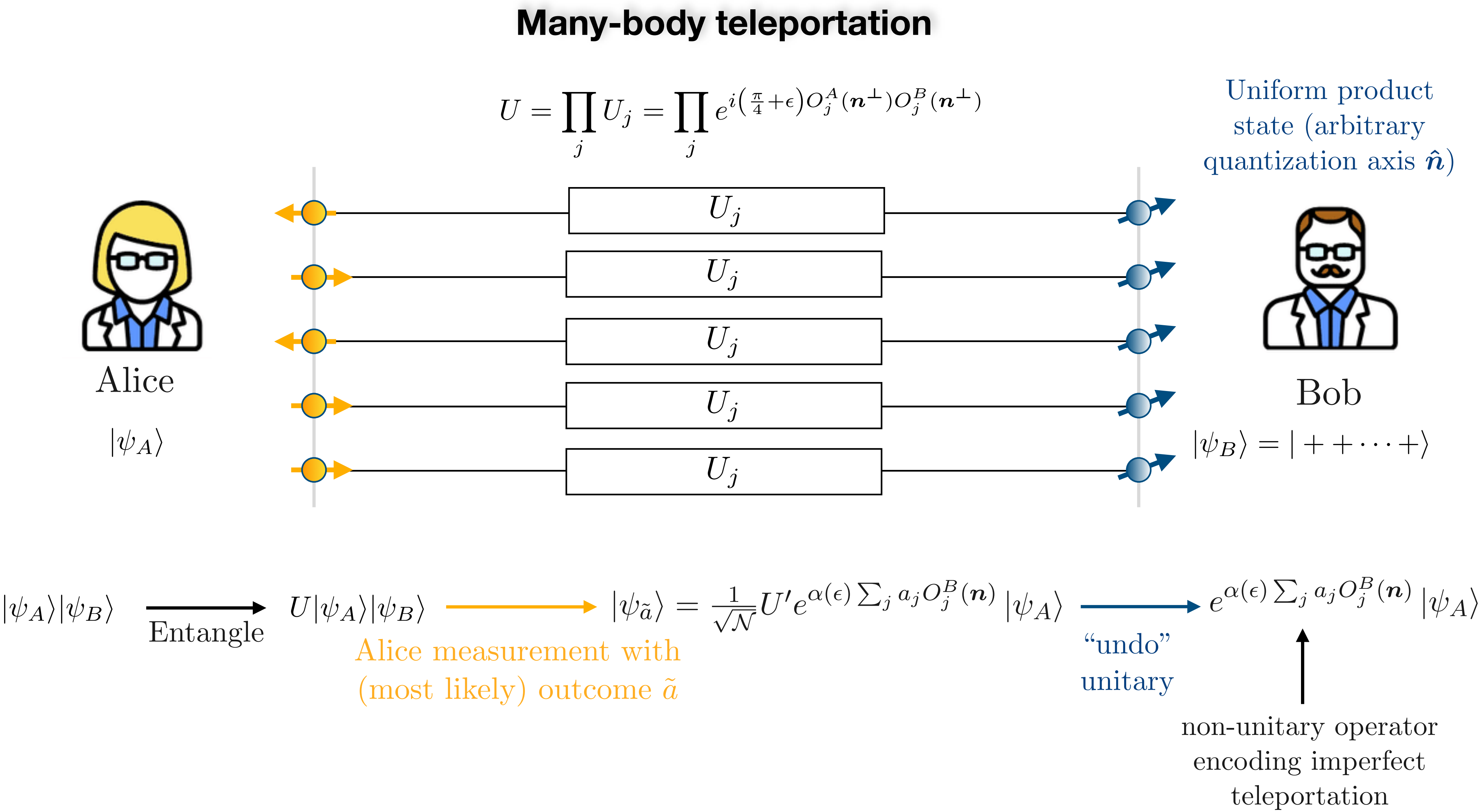


$$|\psi_A\rangle|\psi_B\rangle \xrightarrow{\text{Entangle}} U|\psi_A\rangle|\psi_B\rangle \xrightarrow[\text{with outcome } z_A]{\text{Alice measurement}} |\psi_{z_A}\rangle = U'_{z_A} e^{\alpha(\epsilon)Z_B}|\psi_A\rangle \xrightarrow[\text{unitary}]{\text{“undo”}} \boxed{e^{\alpha(\epsilon)Z_B}}|\psi_A\rangle$$

non-unitary operator encoding imperfect teleportation

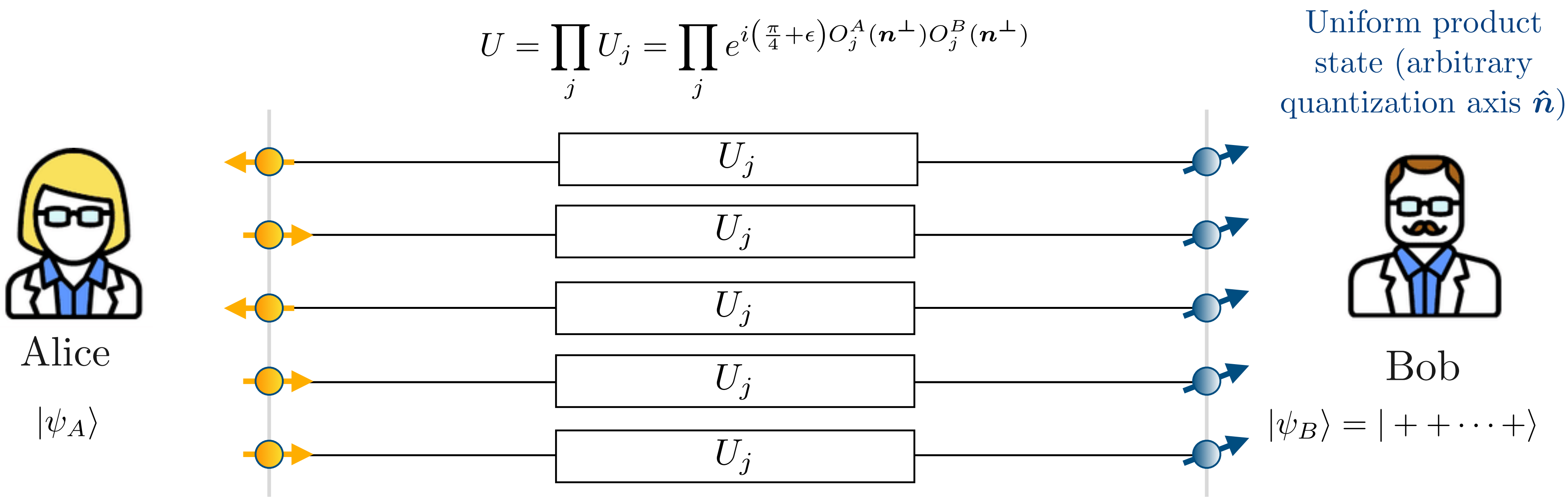
Transfer complete!

But…no one is perfect.


Many-body teleportation
$U = \prod_j U_j = \prod_j e^{i\left(\frac{\pi}{4}+\epsilon\right) O_j^A(\boldsymbol{n}^\perp) O_j^B(\boldsymbol{n}^\perp)}$
Uniform product state (arbitrary quantization axis $\hat{\boldsymbol{n}}$)
U_j
U_j
U_j
U_j
U_j
Alice
$|\psi_A\rangle$
Bob
$|\psi_B\rangle = |++\cdots+\rangle$
$|\psi_A\rangle|\psi_B\rangle$
Entangle
$U|\psi_A\rangle|\psi_B\rangle$
Alice measurement with (most likely) outcome $\tilde{a}$
$|\psi_{\tilde{a}}\rangle = \frac{1}{\sqrt{\mathcal{N}}} U' e^{\alpha(\epsilon)\sum_j a_j O_j^B(\boldsymbol{n})} |\psi_A\rangle$
"undo" unitary
$e^{\alpha(\epsilon)\sum_j a_j O_j^B(\boldsymbol{n})} |\psi_A\rangle$
non-unitary operator encoding imperfect teleportation

Many-body teleportation

$$U = \prod_j U_j = \prod_j e^{i\left(\frac{\pi}{4}+\epsilon\right) O_j^A(\boldsymbol{n}^\perp) O_j^B(\boldsymbol{n}^\perp)}$$



To what extent do protocol imperfections modify entanglement in a teleported quantum state?

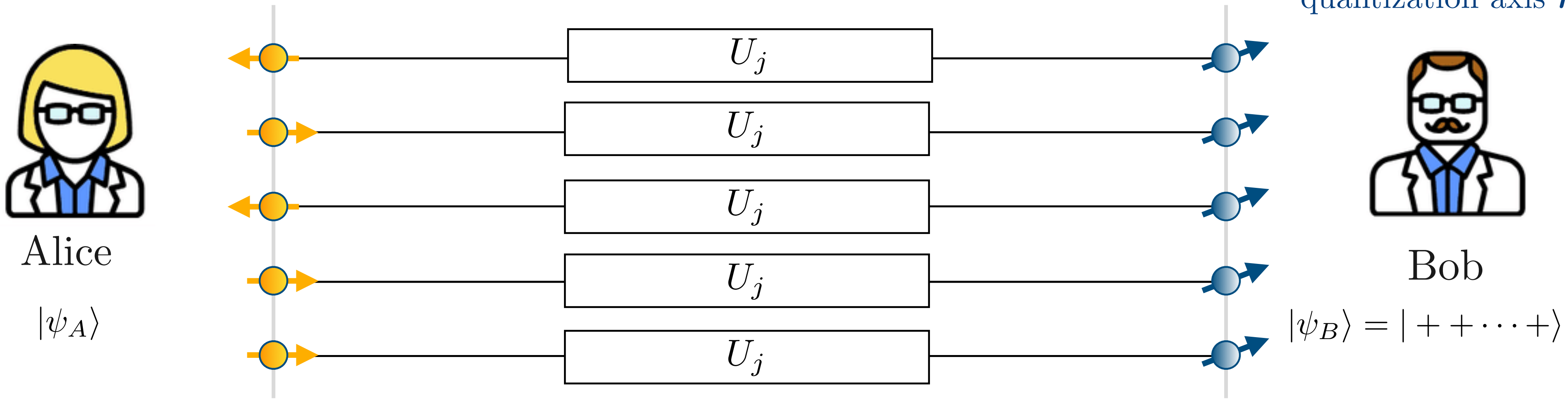
Among the infinite set of possible teleportation protocols, are some more resilient to certain types of imperfections than others?

P. Sala, SM, Y. Liu, J. Alicea, PRX Quantum 5, 030307 (2024).

Many-body teleportation

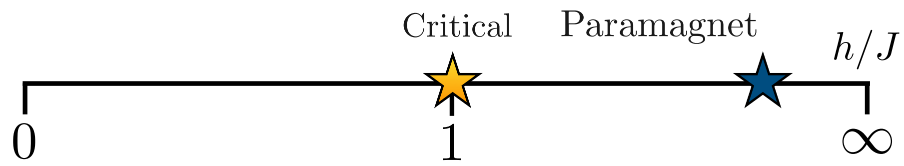
$$U = \prod_j U_j = \prod_j e^{i\left(\frac{\pi}{4}+\epsilon\right)O_j^A(\boldsymbol{n}^\perp)O_j^B(\boldsymbol{n}^\perp)}$$

Uniform product state (arbitrary quantization axis $\hat{\boldsymbol{n}}$)



$|\psi_A\rangle$: ground state of the Ising spin chain

$$H = \sum_j (-JZ_jZ_{j+1} - hX_j)$$



P. Sala, SM, Y. Liu, J. Alicea PRX Quantum 5, 030307 (2024)

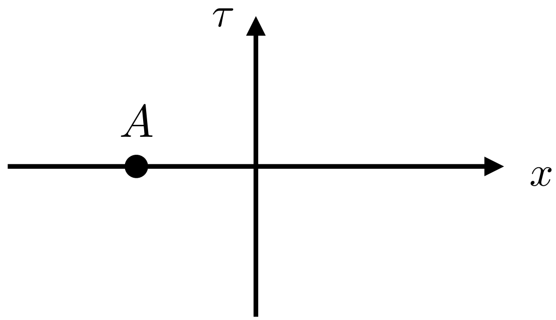
Quantum criticality under imperfect teleportation

Uniform product state (arbitrary quantization axis $\hat{\boldsymbol{n}}$)

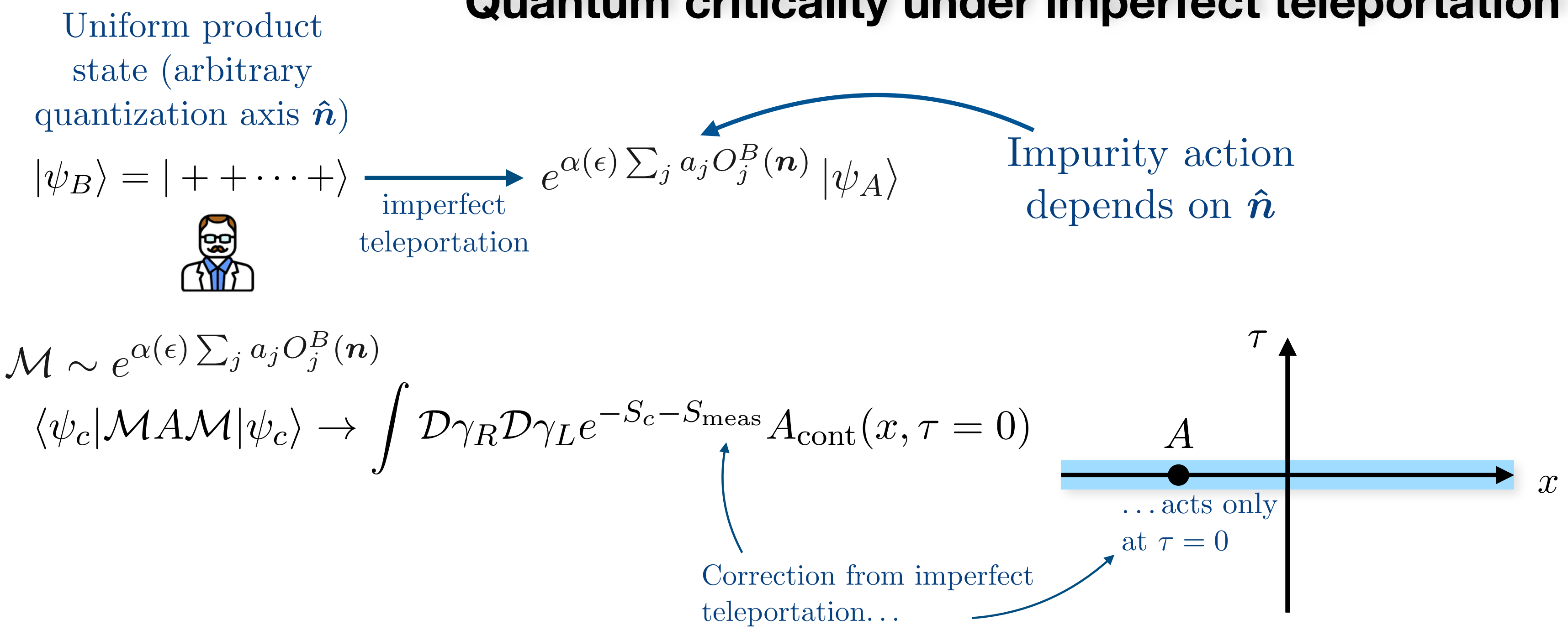
$$|\psi_B\rangle = |++\cdots+\rangle \xrightarrow[\text{teleportation}]{\text{imperfect}} e^{\alpha(\epsilon)\sum_j a_j O_j^B(\boldsymbol{n})}|\psi_A\rangle$$

Impurity action depends on $\hat{\boldsymbol{n}}$

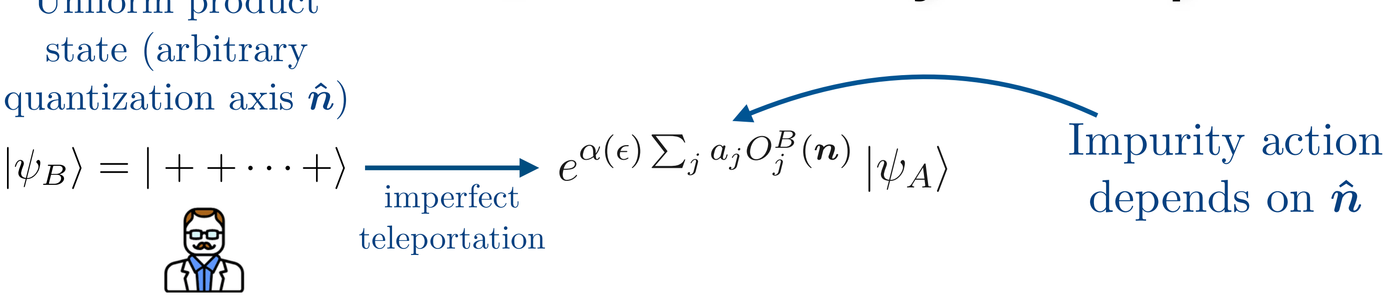
$$\langle\psi_c|A|\psi_c\rangle \to \int \mathcal{D}\gamma_R \mathcal{D}\gamma_L e^{-S_c} A_{\text{cont}}(x, \tau = 0)$$

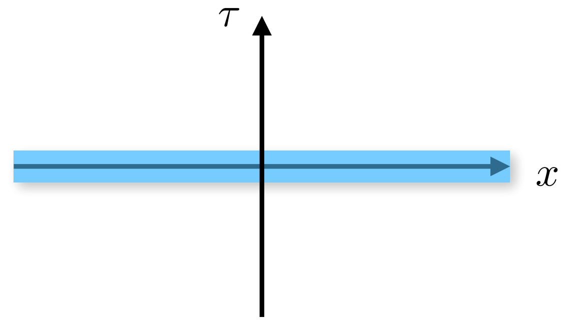


Quantum criticality under imperfect teleportation

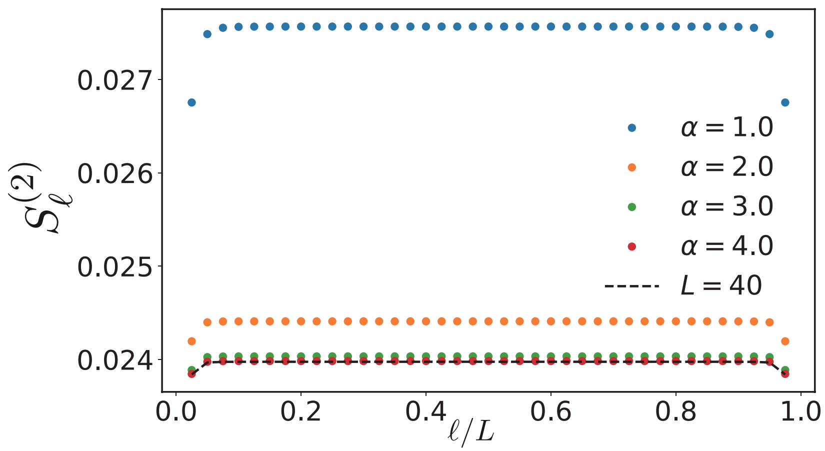


Quantum criticality under imperfect teleportation

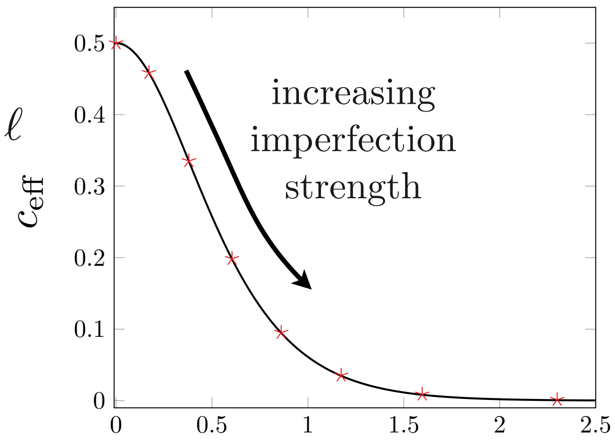




$\hat{\mathbf{n}}$	$\hat{\mathbf{z}}$	$\hat{\mathbf{x}}$	$\hat{\mathbf{y}}$
Effect	Relevant imperfection (bad)	Marginal imperfection (good)	"Marginal" disguised imperfection (better)



$$S_\ell \sim \frac{c}{3}\log\ell \rightarrow \frac{c_{\rm eff}}{3}\log\ell$$



In this talk:

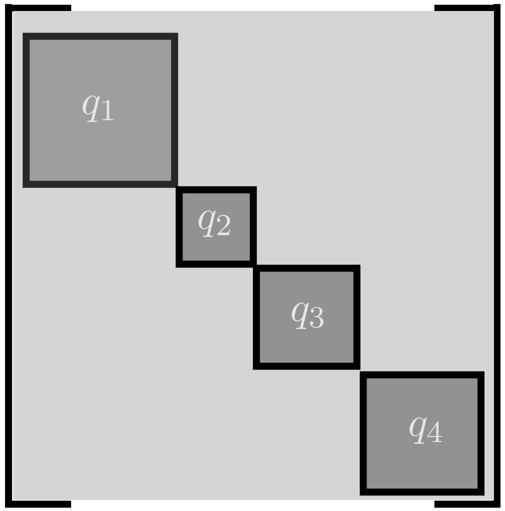
‘30s — ‘60s — last decades — today

Alice, Bob and the quantum ice-cream

Alice and Bob in a symmetric state

$$Q = Q_A \otimes \mathbb{1}_B + \mathbb{1}_A \otimes Q_B$$

$\rho_A =$

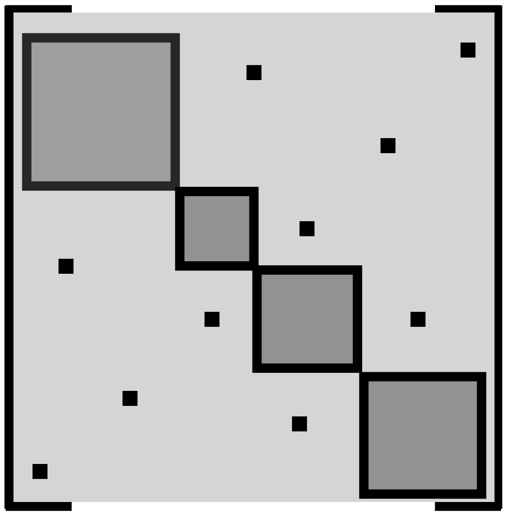


$$[\rho_A, Q_A] = 0$$

M. Goldstein and E. Sela, Phys. Rev. Lett. 120, 200602 (2018)

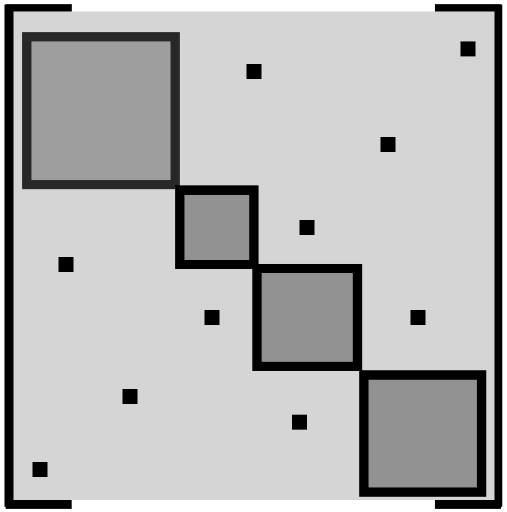
Alice and Bob in an asymmetric state

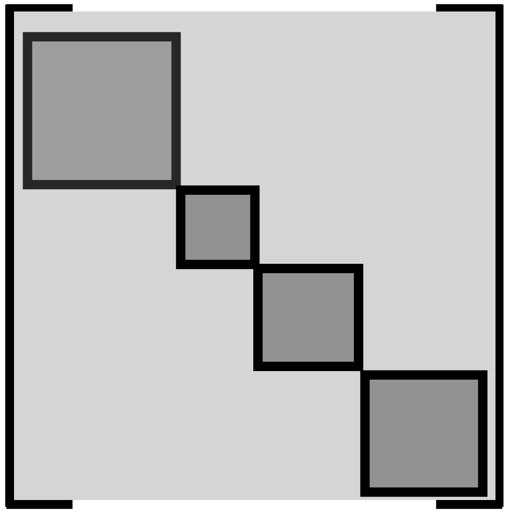
$[\rho_A, Q_A] \neq 0$

$\rho_A =$ 

Entanglement asymmetry as a probe of symmetry breaking

$[\rho_A, Q_A] \neq 0$

$\rho_A =$ 

$\rho_{A,Q} =$ 

$\rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q$

$$\boxed{\Delta S_A^{(n)} = S_A^{(n)}(\rho_{A,Q}) - S_A^{(n)}(\rho_A)}$$

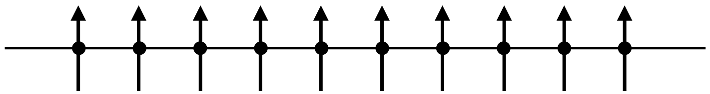
$\Delta S_A^{(n)} \geq 0$

$\Delta S_A^{(n)} = 0 \Leftrightarrow [\rho_A, Q_A] = 0$

F. Ares, SM, P. Calabrese, Nat. Comm. 14, 2036 (2023).

Ferromagnetic state

$$|\Psi\rangle =$$



$$Q = \frac{1}{2}\sum_j Z_j$$

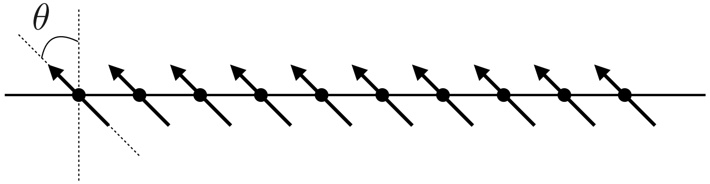
$$[|\Psi\rangle\langle\Psi|, Q] = 0$$

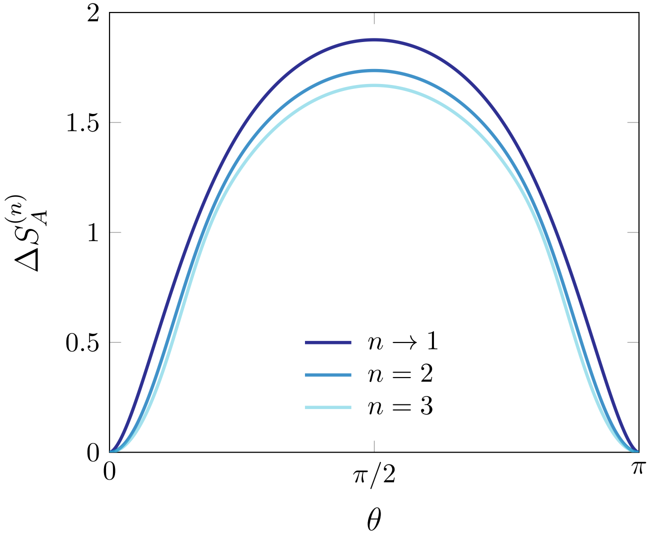
$$\Delta S_A^{(n)} = 0$$

Tilted-ferromagnetic state

$t = 0$

$$|\Psi\rangle =$$





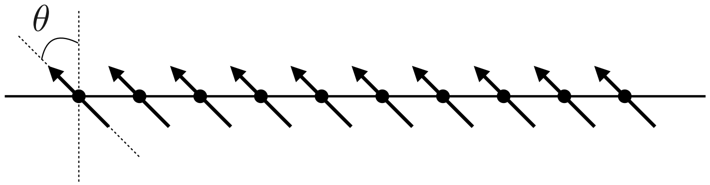
$$Q = \frac{1}{2}\sum_j Z_j$$

$$[|\Psi\rangle\langle\Psi|, Q] \neq 0$$

$$\Delta S_A^{(n)} \neq 0$$

Tilted-ferromagnetic state

$t = 0$

$|\Psi\rangle =$ 

$Q = \frac{1}{2}\sum_j Z_j$

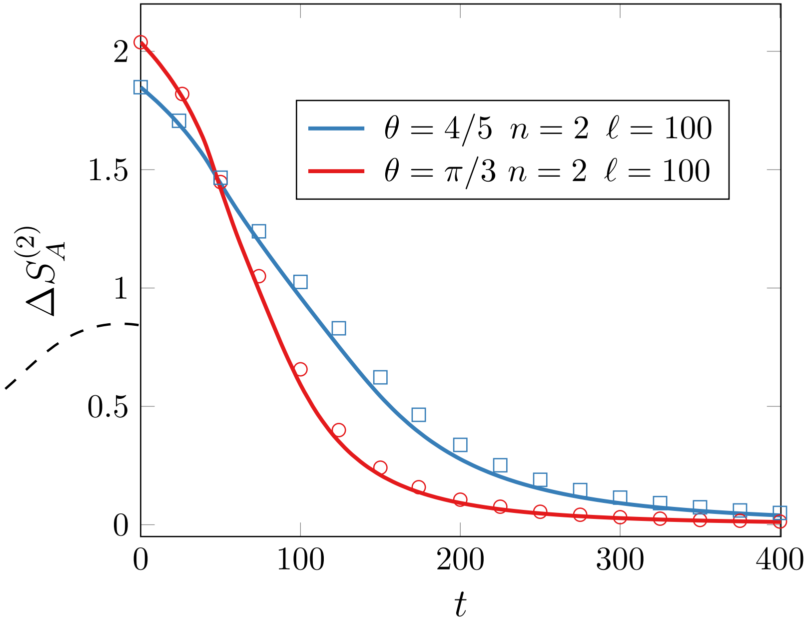
$[|\Psi\rangle\langle\Psi|, Q] \neq 0$

$\Delta S_A^{(n)} \neq 0$

$t > 0$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi\rangle$$

$$H = -\frac{1}{4}\sum_{j=-\infty}^{\infty}[X_jX_{j+1} + Y_jY_{j+1}] \qquad [H, Q] = 0$$



Quantum Mpemba Effect

The more a symmetry is initially broken, the faster it is restored after a global quantum quench to a Hamiltonian preserving it.

A small detour: classical Mpemba effect

Aristotle

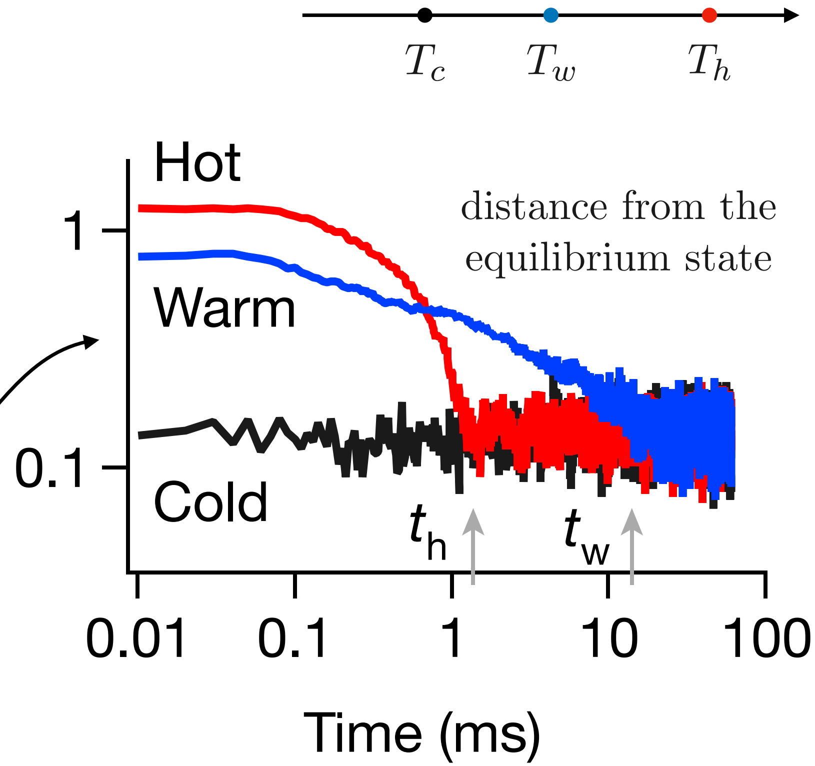
to cool hot water,
begin by putting it in the sun

Meteorologica, 340 BC

Erasto Mpemba

when you put both hot milk and cold milk into a refrigerator at the same time, the hot milk freezes first

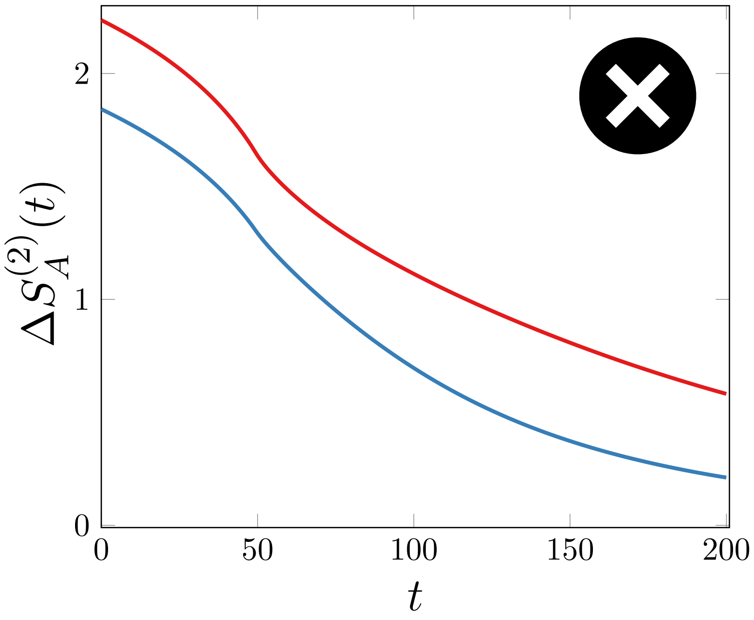
Cool?, 1969



more the system is out-of-equilibrium,
the faster it relaxes

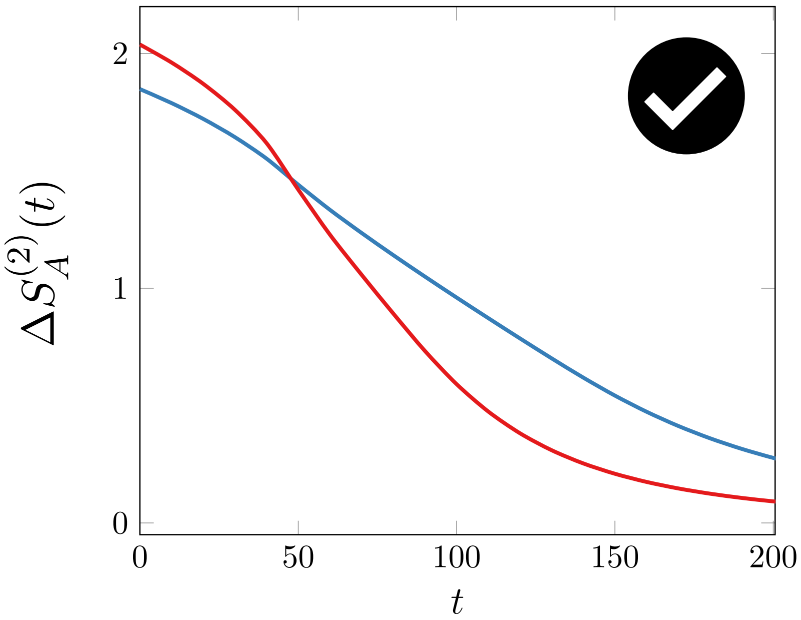
A. Kumar, J. Bechhoefer, Nature, Vol. 584 (2020).

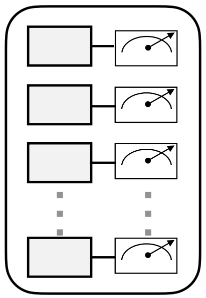
Quantum Mpemba effect in integrable systems



★ $\Delta S_{A,1}^{(2)}(t) < \Delta S_{A,2}^{(2)}(t)$

$t > t_M$





Explanation of the effect in terms of the slowest quasiparticles, which determine the late-time behavior

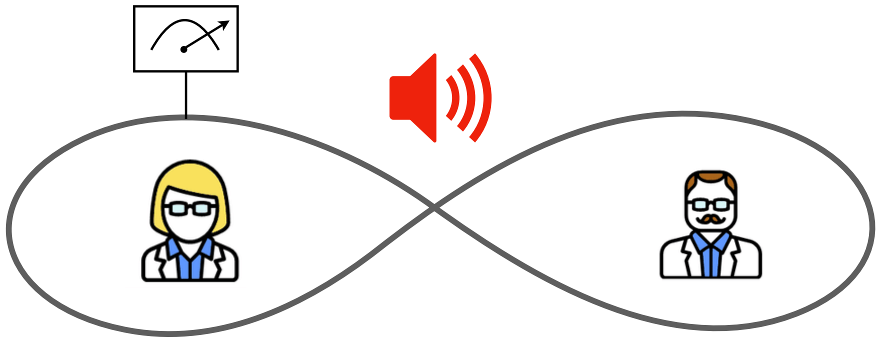
L. Joshi, J. Franke, A. Rath, F. Ares, SM *et al.*, Phys. Rev. Lett. 133, 010402 (2024).

C. Rylands, K. Klobas, F. Ares, P. Calabrese, SM, B. Bertini, Phys. Rev. Lett. 133, 010401 (2024).

Next few years

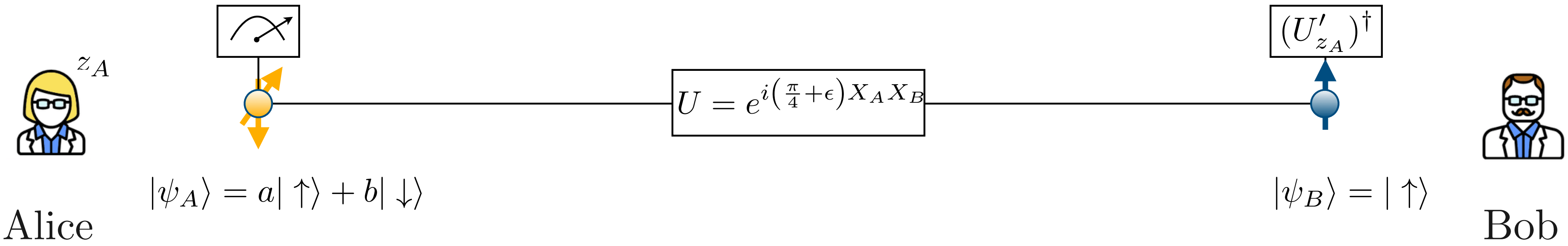
'30s — '60s — last decades — today → ?

Alice, Bob and the quantum ice-cream



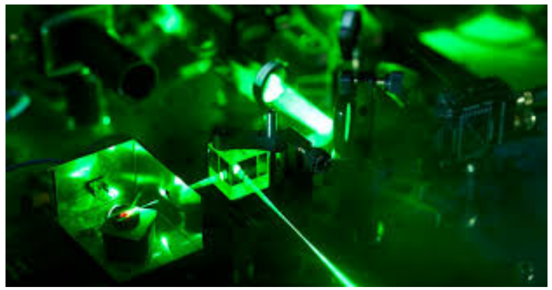
Long-range entanglement under imperfect teleportation

Goal: Transfer Alice's state to Bob without revealing her precise wavefunction



- Teleportation of critical states to several distant parties.

- Criticality used to enhance the performance of quantum sensors and metrological devices.

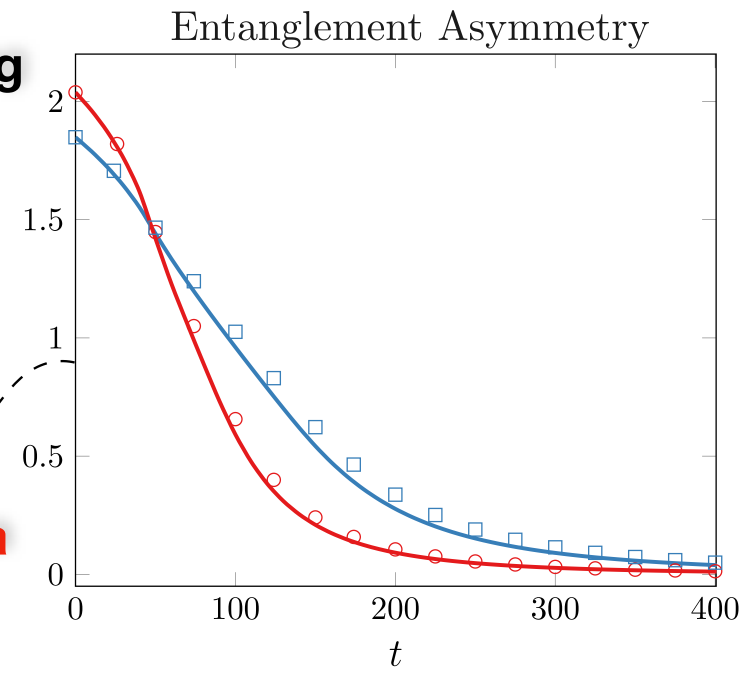


Entanglement asymmetry as a probe of symmetry breaking

Explanation of this phenomenon beyond the integrable case:

- Presence of weak **integrability breaking** terms;
- interplay with **measurements**;
- **chaotic systems** without a quasiparticles description.

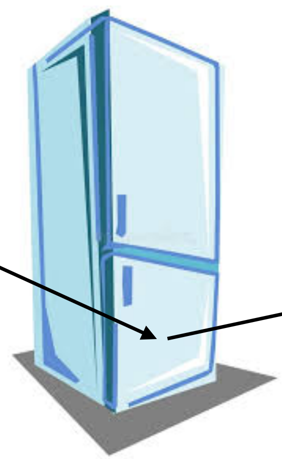
Quantum Mpemba Effect

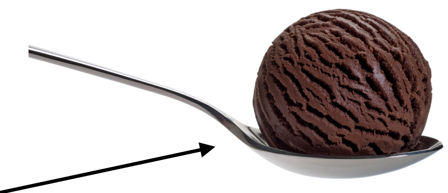


Mpemba discovery:

Hot milk

Cold milk





Thanks!